Bubble Size in Horizontal Pipelines

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Introduction

Bubble sizes for dilute dispersion in horizontal pipelines under turbulent liquid flow conditions are shown to be predicted by a theory proposed by Levich (1962). This theory contains the dependence of dispersed-phase density on the bubble size, which is not included in theories presented by Kolmogoroff (1949) and Hinze (1955). The Kolmogoroff/Hinze and the Levich theories are compared using experimental data from both gas-liquid and liquid-liquid dispersions to show that only the Levich theory can predict both gas bubble and liquid drop sizes with a single constant. Additionally, a generalized equation is proposed that includes the effect of the dispersed-phase viscosity.

Dispersion Theory

The fundamental work in dispersion theory in turbulent flow was conducted independently by both Kolmogoroff and Hinze. They postulated that a maximum stable bubble or drop size, d_{max}, could be determined by the balance between the turbulent pressure fluctuations, tending to deform and break the bubble or drop, and the surface tension force resisting the bubble deformation. The ratio of these forces is defined as the critical Weber number,

$$We_{crit} = \frac{\tau}{\sigma / d_{max}} \tag{1}$$

Levich (1962) postulates a similar force balance, but he considers the balance of the internal pressure of the bubble with the capillary pressure of the deformed bubble. The dispersed-phase density is included through the internal pressure force term, and the capillary pressure is determined from the shape of the deformed bubble rather than the spherical bubble. From this concept the density of the dispersed phase is introduced. A critical Weber number for Levich's theory can be defined as the ratio of these two forces and simplified to the following form:

$$We'_{crit} = \frac{\tau}{\sigma / d_{max}} \left(\frac{\rho_d}{\rho_c}\right)^{1/3} \tag{2}$$

Both the Kolmogoroff/Hinze and Levich theories can be developed in a similar manner to predict the maximum stable bubble or drop size.

The dynamic pressure force of the continuous phase is characterized by Hinze as

$$\tau = \rho_c \overline{v^2} \tag{3}$$

The mean-square spatial fluctuating velocity term, $\overline{v^2}$, describes the turbulent pressure forces of eddies of size d and is defined as the average of the square of the differences in velocity over a distance equal to the bubble or drop diameter. In isotropic homogenous turbulence this velocity term is a function of the energy dissipation rate per unit mass, ϵ. Batchelor (1959) derives this relationship as

$$\overline{v^2} = 2(\epsilon d_{max})^{2/3} \tag{4}$$

Combining the Kolmogoroff/Hinze Weber number with Eqs. 3 and 4 results in the following equation for d_{max}:

$$d_{max} = \left[\left(\frac{We_{crit}}{2}\right)^{0.6}\right]\left(\frac{\sigma}{\rho_c}\right)^{0.6}(\epsilon)^{-0.4} \tag{5}$$

Similarly, Eqs. 3 and 4 can be substituted into the Levich Weber number expression

$$d_{max} = \left[\left(\frac{We'_{crit}}{2}\right)^{0.6}\right]\left(\frac{\sigma^{0.6}}{(\rho_c^2 \rho_d)^{0.2}}\right)(\epsilon)^{-0.4} \tag{6}$$

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These equations describe the maximum bubble size as a function of the local energy dissipated by the turbulence and the physical properties of the fluids. In a pipeline the local energy dissipation rate, ϵ, is equal to the average energy dissipation, $\bar{\epsilon}$, and can be expressed as

$$\bar{\epsilon} = \frac{2fv^3}{D} \tag{7}$$

The friction factor is assumed to be adequately represented by the Blasius relation,

$$f = 0.079Re^{-0.25} \tag{8}$$

Equations 7 and 8 are substituted into Eq. 5

$$d_{max} = 1.38(We_{crit})^{0.6}\left(\frac{\sigma^{0.6}}{\rho_c^{0.5}\mu_c^{0.1}}\right)\left(\frac{D^{0.5}}{v_c^{1.1}}\right) \tag{9}$$

and Eq. 6

$$d_{max} = 1.38(We'_{crit})^{0.6}\left(\frac{\sigma^{0.6}}{(\rho_c^{0.3}\rho_d^{0.2})\mu_c^{0.1}}\right)\left(\frac{D^{0.5}}{v_c^{1.1}}\right) \tag{10}$$

Both equations relate the maximum stable bubble or drop diameter to the pipe diameter and continuous-phase velocity, but differ in their functional dependence on the fluid densities. These equations have previously been used to describe liquid-liquid dispersions in horizontal pipelines, but their application to gas-liquid systems has not been evaluated.

Bubble Size Prediction

The maximum bubble size predictions from the Kolmogoroff/Hinze and Levich theory are compared to experimental data from Holmes (1973) and Holmes and Russell (1975). The physical properties and flow rates of Holmes's system are given in Table 1. Bubble sizes were measured from high-speed motion pictures taken 5.49 m from the introduction of the gas phase. This position in the pipeline was demonstrated by Holmes to correspond to a steady state bubble size. From each run two frames were enlarged and 80 bubble cross-sectional areas were measured using a planimeter. These data were reexamined to determine the maximum diameter and the Sauter mean bubble diameter.

Table 1. Experimental Conditions, Holmes (1973) System

Parameters*				
Continuous-phase viscosity, $\mu_c = 1 \times 10^{-3}$ kg/ms				
Interfacial surface tension, $\sigma = 72 \times 10^{-3}$ N/m				
Dispersed-phase density, $\rho_d = 1.2$ kg/m^3				
Run	D m	v_c m/s	$W_G \times 10^8$ kg/s	H_d m^3/m^3
2	0.0254	4.59	48.5	0.01210
3	0.0254	4.61	4.85	0.00126
4	0.0508	2.72	194.0	0.00687
5	0.0508	3.50	194.0	0.00487
6	0.0254	6.04	48.5	0.00839
7	0.0254	3.44	48.5	0.01654

*Weast, *Handbook of Chemistry and Physics* (1982)

The cross-sectional areas were converted to equivalent spherical diameters and tabulated as cumulative bubble size distributions, shown in Figure 1. These data fit a log normal distribution in which the fractional frequency is given by

$$F(d_i) = \frac{1}{\sqrt{2\pi}\log\sigma_g}\exp\left[-\frac{1}{2}\left(\frac{\log d_i - \log d_g}{\log\sigma_g}\right)^2\right] \tag{11}$$

The geometric mean, d_g, and geometric mean standard deviation, σ_g, were calculated from the raw data and are reported in Table 2. They are defined by,

$$d_g = \left(\prod_{i-1}^{N_T} d_i\right)^{1/N_T} \tag{12}$$

$$\log\sigma_g = \sqrt{\frac{\sum_{i-1}^{N_T}[\log d_i - \log d_g]^2}{N_T}} \tag{13}$$

The width of the dispersion, characterized by σ_g, is fairly constant, and no dependence of velocity on σ_g is apparent. A small dependence on pipe diameter may be present, yielding values for σ_g of 1.45 and 1.30 for pipe diameters of 0.0508 and 0.0254 m, respectively.

Both dispersion theories are based on the maximum size bubble that can exist in a given turbulent flow field. Since the maximum diameter measured was taken from only two photographs for each run, a more reliable estimate of the maximum diameter is obtained from the cumulative size distributions. The diameter that is larger than 99% of all the diameters in the distribution, d_{99}, is chosen to describe the maximum bubble size. The values of both d_{99} and the maximum observed diameter are shown in Table 3. The diameter, d_{99}, corresponds very closely with d_{max} with the exception of run 4.

The dependence of bubble size on D and v can now be confirmed by a linear regression of d_{99} vs. $D^{0.5}/v_c^{1.1}$:

$$d_{99} = (0.0738 \pm 0.0037)(D^{0.5}/v_c^{1.1}) \tag{14}$$

The resulting line, shown in Figure 2, shows a good fit to the data and demonstrates that the theoretical dependence on velocity and pipe diameter has been confirmed. The critical Weber

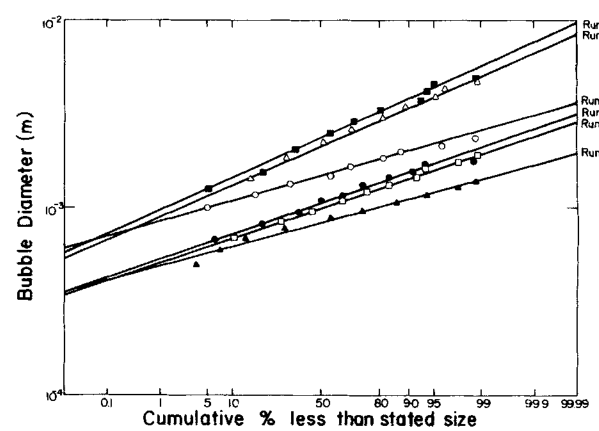

Figure 1. Bubble size distribution.

Table 2. Log Normal Distribution Parameters

Run	$d_g \times 10^3$ (m)	σ_g
2	1.07	1.32
3	0.99	1.31
4	2.31	1.45
5	2.08	1.45
6	0.84	1.27
7	1.45	1.28

number, defined by either Eq. 9 or 10, can be calculated using Eq. 14:

$$We_{crit} = 10.6$$

$$We'_{crit} = 1.1$$

The difference between these two values shows that the fluid properties are not properly accounted for in at least one theory.

For the purposes of mass transfer and heat transfer it is desirable to know the interfacial area of the dispersed phase, which requires a knowledge of the Sauter mean diameter, d_{32}, and is defined as

$$d_{32} = \frac{\sum_{i=1}^{N_T} d_i^3}{\sum_{i=1}^{N_T} d_i^2} \tag{15}$$

The ratio of d_{32} and d_{99} can be determined from the log normal distribution

$$C_n = d_{32}/d_{99} = \exp(2.5 \ln^2 \sigma_g - 2.33 \ln \sigma_g) \tag{16}$$

C_n is a function only of the width of the distribution σ_g, which in turn may have a small dependence on the pipe diameter. The average value of this ratio is

$$C_n = 0.62 \tag{17}$$

This is the same as the value of 0.6 reported by Calabrese et al. (1986a) for liquid-liquid dispersions.

Equation 14 can now be written to describe d_{32} as,

$$d_{32} = 0.0738 C_n \left(\frac{D^{0.5}}{v_c^{1.1}}\right) \tag{18}$$

A plot of Eq. 18, using an average value of C_n (0.62), and the experimental values of d_{32} is given in Figure 2.

Table 3. Characteristic Diameters

Run	Diameter $\times 10^3$ (m)		
	d_{max}	d_{99}	d_{32}
6	1.35	1.45	0.939
3	1.87	1.86	1.19
2	1.81	2.05	1.27
7	2.49	2.56	1.67
5	4.98	4.91	2.88
4	4.92	5.49	3.13

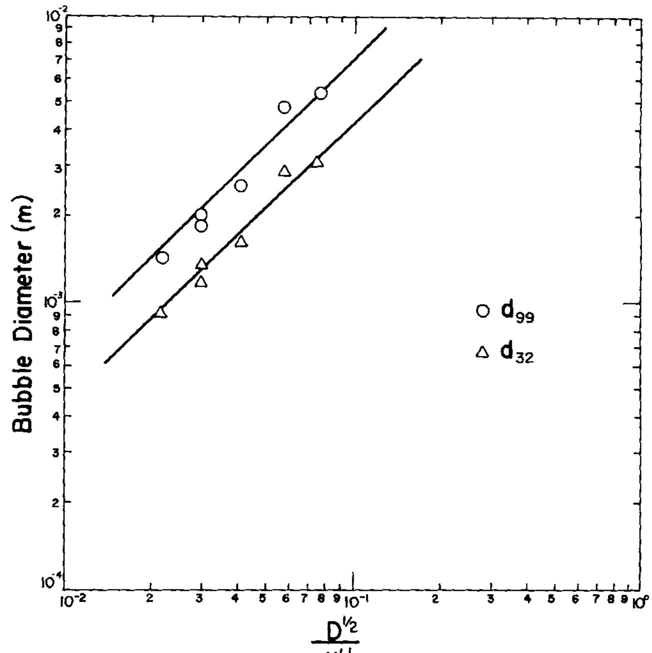

Figure 2. Comparison of bubble size with dispersion theory.

Comparison with Liquid-Liquid Dispersions

The range of applicability of either the Kolmogoroff/Hinze or the Levich theory can be determined by a comparison with liquid drop size data. Liquid-liquid drop size distributions in agreement with the Kolmogoroff/Hinze theory have been reported by Karabelas (1978) and Kubie and Gardner (1977). Maximum liquid drop sizes in disagreement with the Kolmogoroff/Hinze theory and the data of Karabelas and of Kubie and Gardner have been reported by Sleicher (1962) and by Paul and Sleicher (1965). This disagreement may be due to the experimental acceleration technique employed by the authors and is discussed by Karabelas and by Kubie and Gardner.

A summary of the physical properties of the fluids used by Karabelas and Kubie and Gardner is given in Table 4. The pipeline diameters used by Karabelas and Kubie and Gardner were 0.0504 and 0.0172 m, respectively. Critical Weber numbers were calculated from their data, shown in Figure 3, using Eqs. 9 and 10, and are presented in Table 5.

The critical Weber number should depend only on the mechanism of breakup, not on the fluid physical properties. For the liquid-liquid dispersions, the values of We_{crit} calculated from the Kolmogoroff/Hinze theory are a factor of 10 smaller than the value obtained for the gas-liquid dispersion. This discrepancy implies that the breakup mechanism of drops and bubbles is different. Yet, photographs from Holmes (1973) show that bubbles break up by forming a dumbell shape, similar to that observed by Collins and Knudson (1970) in liquid-liquid dispersions. Therefore the values of We_{crit} for liquid-liquid and gas-liquid dispersions should be equal. The theory presented by Levich gives this result, since the values of the We'_{crit} for both liquid-liquid and gas-liquid dispersions are close to one. Therefore the Levich theory is able to describe the breakup of the dispersed phase for both gas and liquid physical properties in a turbulent liquid. This is demonstrated by a regression of the experimental

Table 4. Liquid-Liquid Dispersion Physical Properties

Investigator	Continuous Phase	Dispersed Phase	$\mu_c \times 10^3$ kg/m/s	$\sigma \times 10^3$ N/m	$\rho_c \times 10^{-3}$ kg/m^3	$\rho_d \times 10^{-3}$ kg/m^3
Kubie and Gardner (1977)	*n*-Butyl acetate	Water	0.7	14.5	0.884	0.998
	Water	*n*-Butyl acetate	1.0	14.5	0.998	0.884
	Isoamyl alcohol	Water	4.8	4.86	0.828	0.998
	Water	Isoamyl alcohol	1.0	4.86	0.998	0.828
Karabelas (1978)	Transformer oil	Water	15.6	34.0	0.892	1.0
Holmes (1973)	Water	Air	1.0	72.0	1.0	0.0012

data with Eq. 10 shown in Figure 4. The We'_{crit} obtained from the regression is 1.1. The data cover a large range of physical properties, from 72×10^{-3} to 5×10^{-3} N/m for surface tension, 1×10^{-3} to 16×10^{-3} kg/m · s for continuous-phase viscosity, and 1 to 10^3 kg/m^3 for dispersed-phase density.

A General Equation

Equation 6 is a general equation for prediction of bubble or drop size in turbulent flow fields. It includes all the variables needed to describe turbulent dispersion except for the dispersed-phase viscosity. The viscous forces within the bubble or drop increase its stability, and become important when they are the same order of magnitude as the surface tension forces. Hinze originally included a viscosity grouping in the critical Weber number. This concept has been expanded by Calabrese et al. (1986a) and can be combined with Eq. 6 to include all salient physical fluid properties required to describe bubble or drop size in turbulent liquid flow

$$d_{32} = C_n \left(\frac{We'_{crit}}{2}\right)^{0.6} \left[\frac{\sigma^{0.6}}{(\rho_c^2 \rho_d)^{0.2}}\right] (\epsilon)^{-0.4} (1 + BN_{Vi})^{0.6} \quad (19)$$

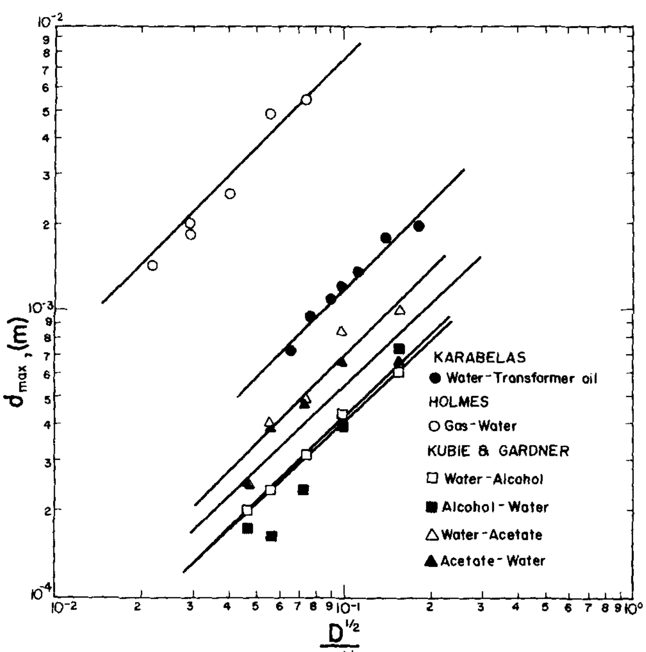

Figure 3. Comparison of bubble and drop size with dispersion theory.

The viscosity number is defined as

$$N_{Vi} = \left[\frac{\mu_d (\epsilon d_{32})^{1/3}}{\sigma}\right] \left(\frac{\rho_c}{\rho_d}\right)^{1/2} \quad (20)$$

For nearly invisid fluids N_{Vi} is small and Eq. 19 reduces to Eq. 6.

The value of We'_{crit} should be 1 if the forces tending to break up and the forces tending to restore the bubble or drop are equal. From Davies' (1986) review of liquid-liquid dispersion data the value of We'_{crit} is calculated and ranges between 0.6 to 1.7 for $\bar{\epsilon}$ ranging from 1 to 1×10^9 W/kg. The Davies study included experimental data from agitated vessels, colloid mills, liquid whistles, valve homogenizers, and ultrasonic homogenizers. The value for pipelines is shown to be 1 in this paper, Table 5. The data of Berkman and Calabrese (1985) for motionless mixers also give a We'_{crit} of 1.

Using Davies' value of $\bar{\epsilon}$ and the data of Calabrese et al. (1986b) for agitated vessels, the value of B in Eq. 19 is calculated to be 1.1. The value of B for motionless mixers is 1.5 (Berkman and Calabrese, 1985). The value of B obtained from the motionless mixer data should be applicable to pipelines in general.

Certain conditions need to be satisfied in order to use Eq. 19 to predict bubble or drop size in a turbulent flow field. The energy dissipation needs to be quantified for the specific fluid flow field surrounding the bubble or drop. For pipelines this was demonstrated in the derivation of Eq. 10. Energy dissipation expressions have also been developed for stirred tanks by Rushton et al. (1950) and Davies (1986). Davies defines the value of $\bar{\epsilon}$ to be the power input into one-half the volume swept out by the impeller blade,

$$\bar{\epsilon} = \frac{N_P N^3 L^5}{\frac{1}{8}(\pi L^2 W)} \quad (21)$$

Table 5. Critical Weber Numbers for Pipe Flow

Investigator	Continuous Phase	Dispersed Phase	We_{crit}	We'_{crit}
Kubie and Gardner	*n*-Butyl acetate	Water	0.9	0.9
	Water	*n*-Butyl acetate	0.6	0.6
	Isoamyl alcohol	Water	1.4	1.5
	Water	Isoamyl alcohol	1.3	1.2
Karabelas	Transformer oil	Water	1.5	1.6
Holmes	Water	Air	10.6	1.1

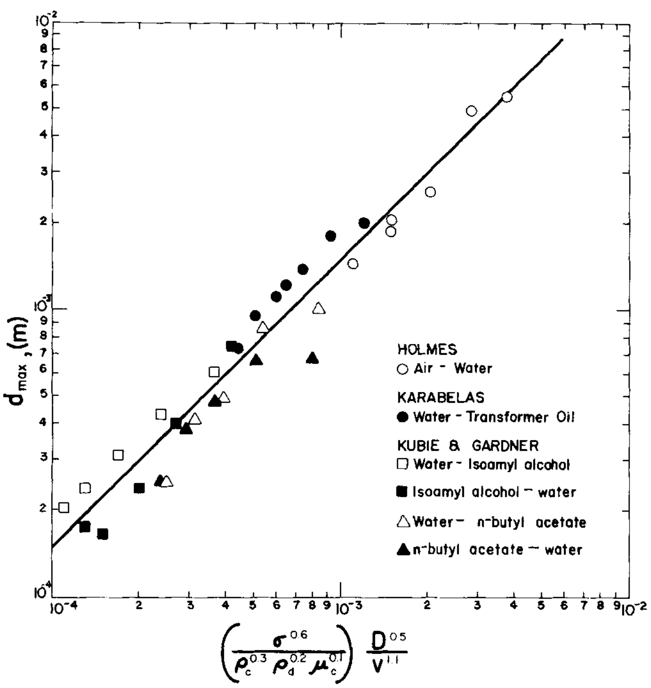

Figure 4. Bubble and drop size compared with Eq. 10.

The system must be noncoalescing, and this typically implies a low dispersed-phase holdup. The residence time of the dispersed phase in the turbulent field must be sufficient to reach the steady state size distribution. In a pipeline, data from Holmes (1973) show this to be between 1 to 2 s at continuous-phase velocities between 2.5 and 5 m/s. The predicted value of d_{max} can not be less than the minimum bubble or drop size that can be produced by the turbulent flow field. Levich postulates that the minimum bubble or drop size is reached when the local flow around the bubble or drop is such that the turbulence is no longer able to deform the drop or bubble. The minimum bubble or drop size is not encountered under most processing situations, since it is a very small. For example, the minimum drop size for the data given above is on the order of 1 μm.

Notation

B = constant, Eq. 19
C_n = conversion factor d_{32}/d_{99}
d = bubble or drop diameter, m
d_i = individual bubble diameter, m
d_g = geometric mean diameter, m
d_{max} = maximum bubble or drop diameter, m
d_{32} = Sauter mean diameter, m
d_{99} = d > 99% of all diameters in the distribution
f = fanning friction factor
D = inside pipe diameter, m
F = fractional number distribution function
H = holdup, m^3 gas/m^3 pipeline
L = impeller diameter, m
N = impeller speed, rev/s
N_P = power number, $P/(\rho_c N^3 L^5)$
N_T = total number of bubbles in the distribution
N_{Vi} = bubble viscosity number
N_{We} = bubble Weber number $\tau/\sigma/d$
P = power, W/s
p = pressure, Pa
Re = Reynolds number, $(\rho_c v_c D)/\mu_c$
v = superficial velocity, m/s
$\overline{v^2}$ = $\overline{[v(x+d,t) - v(x,t)]^2}$, m^2
x = multidirection Cartesian coordinate, m
W = impeller blade width, m
We_{crit} = critical Weber number for Kolmogoroff/Hinze development
We'_{crit} = critical Weber number for Levich development
W_G = gas mass flow rate, kg/m^3

Greek letters

ϵ = local energy dissipation rate per unit mass, W/kg
$\bar{\epsilon}$ = ϵ averaged over vessel volume where dispersion is formed
μ = viscosity, kg/m · s
ρ = density, kg/m^3
σ = interfacial surface tension, N/m
σ_g = geometric mean standard deviation
τ = dynamic pressure fluctuations, N/m^2

Subscripts

c = continuous phase
d = dispersed phase

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